FUZZY BLACK TOP-HAT AND HIT-OR-MISS TRANSFORMATIONS AND THEIR APPLICATIONS

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Introduction

Fuzzy mathematical morphology, within the framework of Soft Computing, has proved to be a powerful tool to handle imprecision in images. This theory provides competitive results positioning it in the state-of-the-art of many applications.

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Fuzzy mathematical morphology, within the framework of Soft Computing, has proved to be a powerful tool to handle imprecision in images. This theory provides competitive results positioning it in the state-of-the-art of many applications.

This theory relies on the use of fuzzy morphological operators defined using fuzzy conjunctions and fuzzy implication functions.

Definition

An increasing binary operator $C : [0,1]^2 \to [0,1]$ is a *fuzzy conjunction* whenever it is increasing in both variables and it satisfies $C(0,1) = C(1,0) = 0$ and $C(1,1) = 1$.

Definition

A binary operator $I : [0,1]^2 \to [0,1]$ is a *fuzzy implication function* if it is decreasing in the first variable, increasing in the second one and it holds that $I(0,0) = I(1,1) = 1$ and $I(1,0) = 0$.

Definition

Let C be a fuzzy conjunction and I be a fuzzy implication function. The fuzzy dilation $\mathcal{D}_C(A, B)$ and the fuzzy erosion $\mathcal{E}_I(A, B)$ of a grey-scale image A and a grey-scale structuring element B are defined as:

$$\mathcal{D}_C(A, B)(y) = \sup_{x \in d_A \cap T_y(d_B)} C(B(x-y), A(x)),$$

$$\mathcal{E}_I(A, B)(y) = \inf_{x \in d_A \cap T_y(d_B)} I(B(x-y), A(x)),$$

where d_A and d_B denote the definition domains of A and B and $T_y(d_B)$ is the translation of the fuzzy set d_B by vector $y \in \mathbb{R}^2$ given by $T_y(d_B)(z) = d_B(z-y)$.

INTRODUCTION

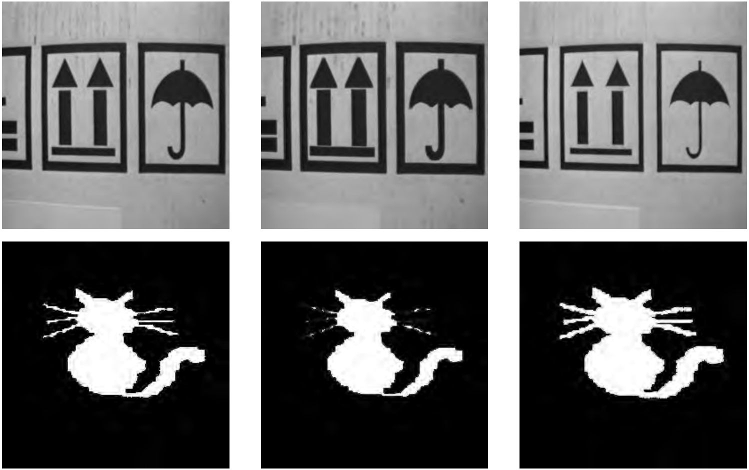

Figure: From left to right, original image, fuzzy erosion and fuzzy dilation.

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- *The Fuzzy Morphological Hit-or-Miss.*
- *The Fuzzy Top-Hat.*

In this presentation, the main goals are:

1. To provide a general overview of the recent research on the *Fuzzy Top-Hat* and the *Fuzzy Morphological Hit-or-Miss*.
2. To present their definitions and theoretical properties.
3. To present their applications in several fields such as:
   - Curvilinear object detection.
   - Eye fundus vessels segmentation.
   - Hair skin removal in dermoscopic imagery

Fuzzy Morphological Hit-or-Miss

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$$A \circledast B = \{x \ : \ (B_{FG})_x \subseteq A, \ \ (B_{BG})_x \subseteq A^c\} = (A \ominus B_{FG}) \cap (A^c \ominus B_{BG}).$$

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Definition

Let N be a strong fuzzy negation, I a fuzzy implication function and C a fuzzy conjunction. The *fuzzy Hit-or-Miss transform* (FHMT) of the grey-level fuzzy image A with respect to the grey-scale structuring element $B = (B_1, B_2)$ is defined by

$$\mathtt{FHMT}_{C,I,N}(A, B)(y) = C\left(\mathcal{E}_I(A, B_1)(y), \mathcal{E}_I(N(A), B_2)(y)\right),$$

where $N(A)(x) = N(A(x))$ for all $x \in d_A$ and $C(A, B)(x) = C(A(x), B(x))$, for all $x \in d_A \cap d_B$.

Theorem

Let A be a binary image and $B = (B_1, B_2)$ be a binary structuring element. Then the FHMT *operator coincides with the classical binary Hit-or-Miss, i.e.,* $\mathtt{FHMT}_{C,I,N}(A, B) = A \circledast B$.

Moreover, it is invariant for translations and monotone in the sense of the next result.

Proposition

*Let T be a t-norm, I a fuzzy implication function satisfying $I(x,y)=1$ iff $x \leq y$ **(OP)**, N a fuzzy negation, A a grey-scale image and $B_1^* = (B_1, N(B_1))$ and $B_2^* = (B_2, N(B_2))$ two grey-scale structuring elements. Then it holds that:*

i) $\mathtt{FHMT}_{T,I,N}(A, B_2^*)(y) \leq \mathtt{FHMT}_{T,I,N}(A, B_1^*)(y)$ *for all* $x \in d_{T_y(B_1)} \cap d_{T_y(B_2)}$ *and* $y \in \mathbb{R}^n$ *whenever* $A(x) \leq B_1(x-y) \leq B_2(x-y)$.

ii) $\mathtt{FHMT}_{T,I,N}(A, B_2^*)(y) \leq \mathtt{FHMT}_{T,I,N}(A, B_1^*)(y)$ *for all* $x \in d_{T_y(B_1)} \cap d_{T_y(B_2)}$ *and* $y \in \mathbb{R}^n$ *whenever* $B_2(x-y) \leq B_1(x-y) \leq A(x)$.

It is able to detect all the parts of the image which are equivalent to the structuring element with value 1.

Proposition

Let A be a grey-scale image, $B = (B_1, N(B_1))$ a grey-scale structuring element, T a t-norm, I a fuzzy implication function satisfying (OP) *and $y \in \mathbb{R}^n$. Then B_1 is a part of A at the point y iff $\mathrm{FHMT}_{T,I,N}(A, B)(y) = 1$.*

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as a "degree of similarity" related with the value of the aggregation operator C at the point (u,v), where

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as a "degree of similarity" related with the value of the aggregation operator C at the point (u, v), where
  - u shows how B_1 is included in the grey-level image A,
  - v shows how B_2 included in $N(A)$.

So, if the value is closer to 1, it is more certain that the shape of B_1 is contained in the image A.

APPLICATIONS

A TOY EXAMPLE

Figure: Fuzzy morphological hit-or-miss transform detection of E’s.

We have recently proposed a *curvilinear object detector based on the fuzzy morphological hit-or-miss transform* defined as:

$$D_1(A) = \operatorname*{Agg_2}_{s \in \{s_1,\ldots,s_n\}} \left\{ \operatorname*{Agg_1}_{\alpha \in \{\alpha_1,\ldots,\alpha_m\}} \left\{ \mathtt{FHMT}_{C,I,N}(A, B_1^{(s,\alpha)}, B_2^{(s,\alpha)}) \right\} \right\},$$

where $s_1, \ldots, s_n$ and $\alpha_1, \ldots, \alpha_m$ denote the sizes and orientations of the structuring elements $B_1^{(s,\alpha)}$ and $B_2^{(s,\alpha)}$.

APPLICATIONS

A CURVILINEAR OBJECT DETECTOR

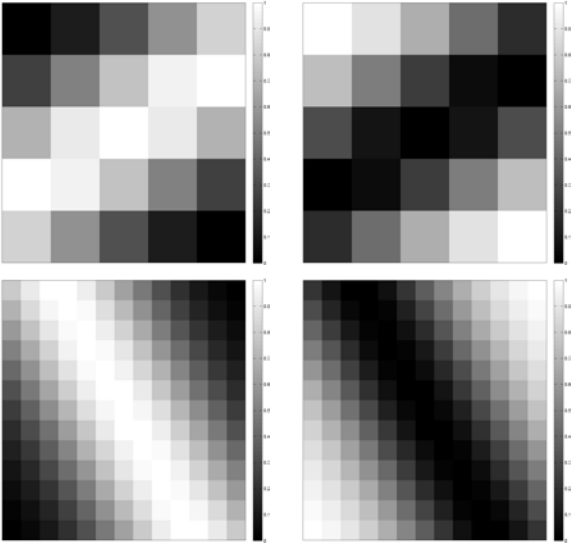


Figure: Sample of the structuring element pairs of the hit-or-miss transform, with

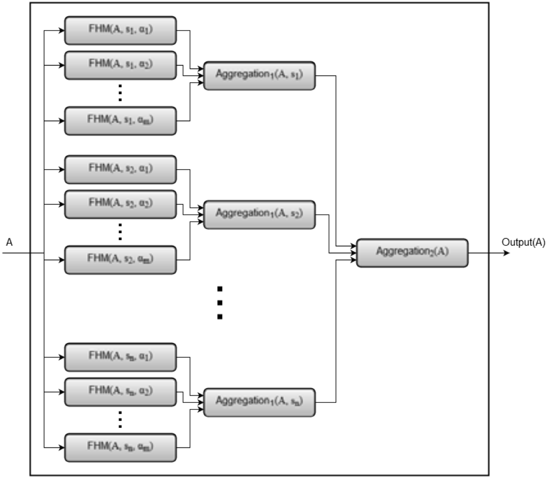


Figure: Workflow of the hit-or-miss curvilinear object detector applied at scales $\{s_1, \ldots, s_n\}$ and orientations $\{\alpha_1, \ldots, \alpha_m\}$.

APPLICATIONS

A CURVILINEAR OBJECT DETECTOR

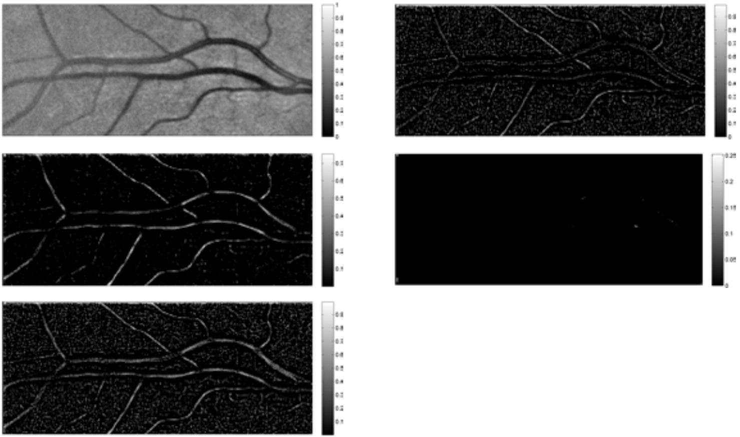


Figure: Results obtained with 6 orientations and different sizes $\{3, 5\}$, $\{5, 9\}$, $\{9, 13\}$ and $\{3, 5, 9, 13\}$.

Figure: Results obtained with structuring elements of sizes $s \in \{5, 9, 13\}$ and different number of orientations $\{2, 4, 6, 8\}$.

Top Hat transform

TOP HAT – DEFINITION

Definition

Let C be a conjunction, I a fuzzy implication function, and A, B grayscale images. Then, the *Opening*, $\mathcal{O}_{C,I}(A,B)$; and the *Closing*, $\mathcal{C}_{C,I}(A,B)$; are:

$$\mathcal{O}_{C,I}(A,B) = \mathcal{D}_C(\mathcal{E}_I(A,B),\bar{B}),$$
$$\mathcal{C}_{C,I}(A,B) = \mathcal{E}_I(\mathcal{D}_C(A,B),\bar{B}),$$

where $\bar{B}(x) = B(-x)$.

Definition

Let C be a conjunction, I a fuzzy implication function, and A, B grayscale images. Then, the *White Top-Hat* transform, $\mathtt{WTH}_{C,I}(A,B)$; and the *Black Top-Hat* transform, $\mathtt{BTH}_{C,I}(A,B)$; are:

$$\mathtt{WTH}_{C,I}(A,B) = A - \mathcal{O}_{C,I}(A,B),$$
$$\mathtt{BTH}_{C,I}(A,B) = \mathcal{C}_{C,I}(A,B) - A.$$

TOP HAT – BEHAVIOUR

Original image

TOP HAT – BEHAVIOUR

Erosion

Original image

Dilation

TOP HAT – BEHAVIOUR

Erosion

Original image

Dilation

Opening

Closing

TOP HAT – BEHAVIOUR

Opening

Original image

Closing

TOP HAT – BEHAVIOUR

Opening

Original image

Closing

White top-hat

Black top-hat

TOP HAT – BEHAVIOUR

Opening

Original image

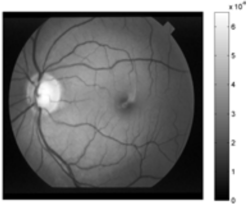
Closing

White top-hat

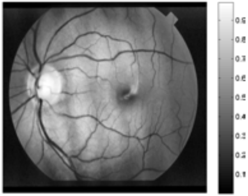
Avrg top-hats

Black top-hat

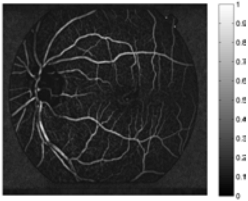
TOP HAT – CURVILINEAR DETECTOR



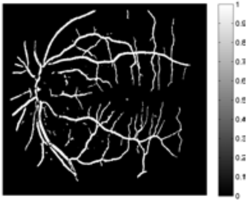
(a) Green Channel from Eye-Fundus



(b) CLAHE

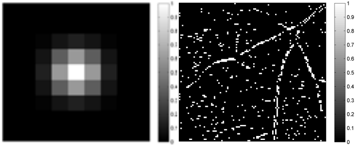


(c) Top Hat

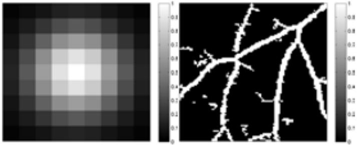


(d) Hysteresis

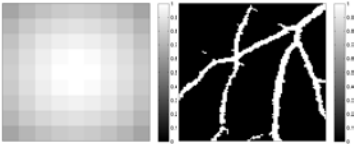
TOP HAT – CURVILINEAR DETECTOR



(a) $\sigma^2 = 1.0$.



(b) $\sigma^2 = 4.2$.

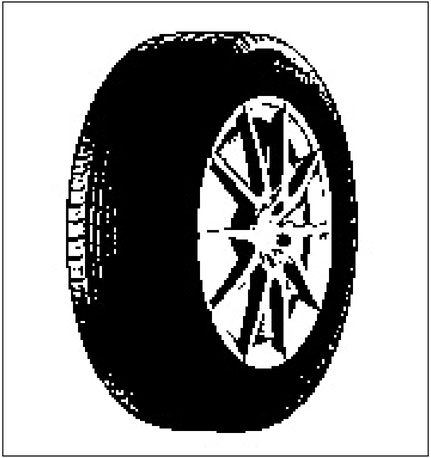


(c) $\sigma^2 = 75.0$.

Color Top Hat transform

FOREGROUND VS BACKGROUND

What is the object?



FOREGROUND VS BACKGROUND

What is the object?

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FOREGROUND VS BACKGROUND – ALTERNATIVES

$$\mathcal{D}_C(A, B)(y) = \underset{x}{\text{Aggregation}}\left\{ f(A(x), B(x-y)) \right\}. \qquad (1)$$

- Process each channel independently.

FOREGROUND VS BACKGROUND – ALTERNATIVES

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- Process each channel independently.
- Define a certain order among colors (and select maximum).
- *Find* a certain order among colors (and select maximum).
- Consider an *averaging* function for colors.

CIEL*A*B* COLOR SPACE

- 3 channels
  - L: luminance
  - a* i b*: chromatic information.
- Perceptually uniform.

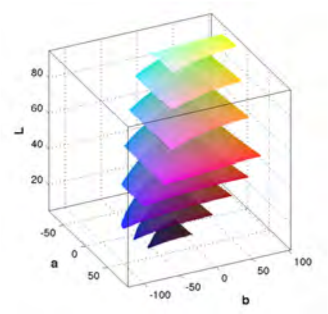

Figure: CIELab gamut

Idea: computing "grayscale morphology" on first channel, and drag the values from other channels.

$$\mathcal{D}_C(A,B)(y) = \Big\{ \Big(C\big(B(x-y), A_1(x)\big),\ A_2(x),\ \ldots,\ A_m(x) \Big) \text{ st. } x \in d_A \cap T_y(d_B) \text{ and } C\big(B(x-y), A_1(x)\big) \text{ is maximum} \Big\}. \tag{2}$$

$$\mathcal{E}_I(A,B)(y) = \Big\{ \Big(I\big(B(x-y), A_1(x)\big),\ A_2(x),\ \ldots,\ A_m(x) \Big) \text{ st. } x \in d_A \cap T_y(d_B) \text{ and } I\big(B(x-y), A_1(x)\big) \text{ is minimum} \Big\}. \tag{3}$$

FUZZY COLOR MORPHOLOGY – BEHAVIOUR

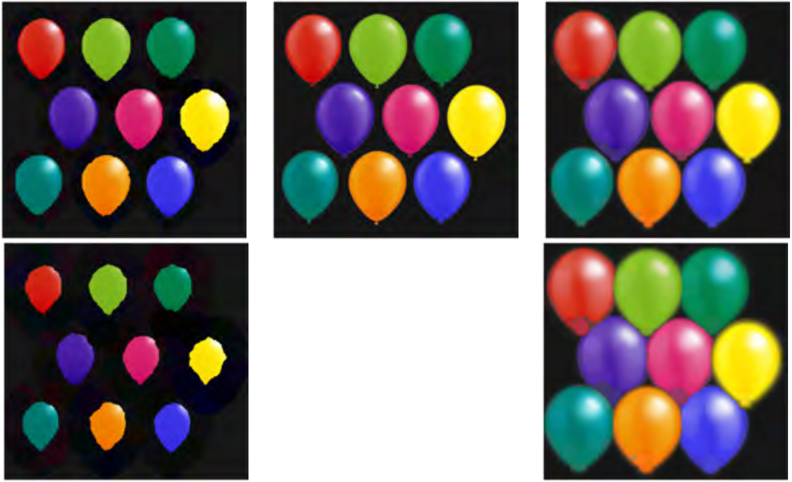

Figure: Erosion (left), and dilation (right) of the *Balloons* image (center), with the minimum t-norm, the Gödel implication and a 15×15-pixel (up) or a 31×31-pixel (down) Gaussian-shaped structuring element.

COLOR TOP HAT – DEFINITION

Definition

Let C be a conjunction, I a fuzzy implication function, and A, B grayscale images. Then, the *White Top-Hat* transform, $\mathtt{WTH}_{C,I}(A, B)$; and the *Black Top-Hat* transform, $\mathtt{BTH}_{C,I}(A, B)$; are:

$$\mathtt{WTH}_{C,I}(A, B) = A - \mathcal{O}_{C,I}(A, B),$$
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$$\mathtt{WTH}^{\equiv}{}_{C,I}(A,B) = d(A, \mathcal{O}^{\equiv}{}_{C,I}(A,B)),$$
$$\mathtt{BTH}^{\equiv}{}_{C,I}(A,B) = d(A, \mathcal{C}^{\equiv}{}_{C,I}(A,B)),$$

where d is a map from pairs of colors to $[0,1]$.

COLOR TOP HAT – BEHAVIOUR

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Erosion

Original image

Dilation

COLOR TOP HAT – BEHAVIOUR

Erosion

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Opening

Closing

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Original image

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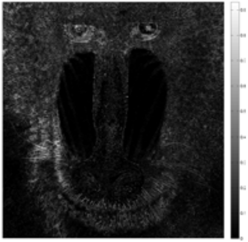
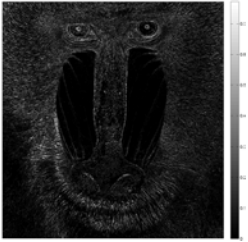
Opening

Original image

Closing



White top-hat

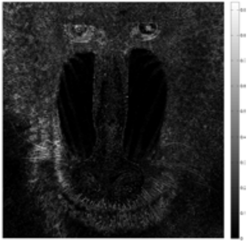


Black top-hat

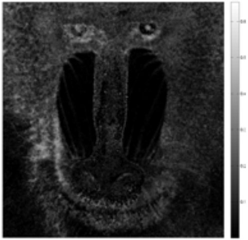
Opening

Original image

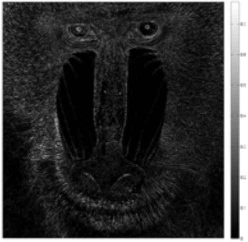
Closing



White top-hat

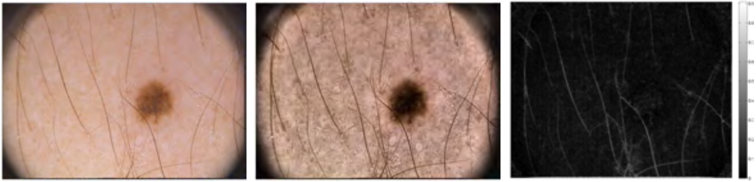


Avrg top-hats



Black top-hat

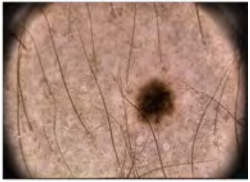
COLOR TOP HAT – CURVILINEAR DETECTOR



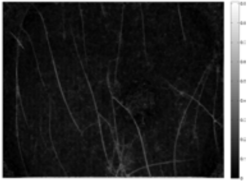
(a) Original (b) Preprocessing (c) Aggregated top-hat

COLOR TOP HAT – CURVILINEAR DETECTOR

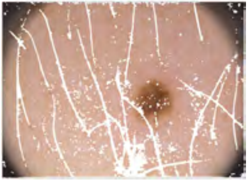
(a) Original



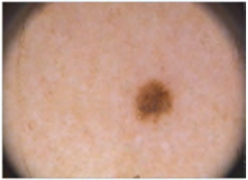
(b) Preprocessing



(c) Aggregated top-hat



(d) Mask to inpaint



(e) Final result

Conclusions

CONCLUSIONS

- Theoretical background of mathematical morphology.
- Development of color/grayscale curvilinear detectors.
- Competitive results in real applications.

Thank you